MA4532: APPLIED FUNCTIONAL ANALYSIS

Effective Term
Semester B 2024/25

Part I Course Overview

Course Title
Applied Functional Analysis

Subject Code
MA - Mathematics
Course Number
4532

Academic Unit
Mathematics (MA)

College/School
College of Science (SI)

Course Duration
One Semester

Credit Units
3

Level
B1, B2, B3, B4 - Bachelor's Degree

Medium of Instruction
English

Medium of Assessment
English

Prerequisites
MA2503 Linear Algebra / MA1503 Linear Algebra with Applications; and
MA2508 Multi-variable Calculus; and
MA3512 Partial Differential Equations / MA3524 Analysis

Precursors
Nil

Equivalent Courses
Nil

Exclusive Courses
Nil

Part II Course Details

Abstract

This course aims at introducing basic topics of Functional Anlaysis with applications to Quantum Mechanics and partial differential equations (PDE’s). Detail topics are the following:
- Hilbert and Banach spaces (Definition and basic properties)
- Operators on Hilbert Spaces
- L2 integrable functions and Fourier transforms
- Spectral Theorem
- Applications to Quantum Mechanics (optional)
- Applications to PDE’s: Fredholm Alternative (optional)

Course Intended Learning Outcomes (CILOs)

	CILOs	Weighting (if app.)	DEC-A1	DEC-A2	DEC-A3
1	explain clearly properties of normed linear spaces and bounded linear operators on such spaces.	15	x		
2	state and prove various structure theorems on Hilbert and Banach spaces.	15		x	
3	understand applications of Hilbert space structures to Quantum Mechanics and/or PDE’s	25	x	x	
4	state the Spectral Theorem and apply it to examples of linear operators	25		x	x
5	the combination of CILOs 1-4	20	x	x	x

A1: Attitude
Develop an attitude of discovery/innovation/creativity, as demonstrated by students possessing a strong sense of curiosity, asking questions actively, challenging assumptions or engaging in inquiry together with teachers.

A2: Ability
Develop the ability/skill needed to discover/innovate/create, as demonstrated by students possessing critical thinking skills to assess ideas, acquiring research skills, synthesizing knowledge across disciplines or applying academic knowledge to real-life problems.

A3: Accomplishments
Demonstrate accomplishment of discovery/innovation/creativity through producing /constructing creative works/new artefacts, effective solutions to real-life problems or new processes.

Learning and Teaching Activities (LTAs)

	LTAs	Brief Description	CILO No.	Hours/week (if applicable)
1	Lectures	Learning through teaching is primarily based on lectures.	1, 2, 3, 4, 5	39 hours in total
2	Take-home assignments	Learning through take-home assignments helps students to understand the basic theory of Hilbert spaces, and apply it to mathematical problems arising from PDE’s and Quantum Mechanics.	1, 2, 3, 4	after-class

Assessment Tasks / Activities (ATs)

	ATs	CILO No.	Weighting (%)	Remarks (e.g. Parameter for GenAI use)
1	Test / Midterm	1, 2, 3, 4	20	Questions are designed for the first part of the course to see how well the students have learned basic properties of Hilbert spaces and recognized the applications of the projection theorem in function-theoretic approximation.
2	Quizzes	1, 2, 3, 4	20	These are skills based assessment to help students understand structural properties of Hilbert spaces, the Fredholm alternative and the associated results with applications.
3	Formative take-home assignments	1, 2, 3, 4	0	The assignments provide students chances to demonstrate their achievements in applying techniques of functional analysis learned from this course.

Continuous Assessment (%)

40

Examination (%)

60

Examination Duration (Hours)

2

Additional Information for ATs

40% Coursework 60% Examination (Duration: 2 hours, at the end of the semester) For a student to pass the course, at least 30% of the maximum mark for the examination must be obtained.

Assessment Rubrics (AR)

Assessment Task

1. Test / Midterm

Criterion

Ability to understand the basic concepts and applications of Functional Analysis

Excellent (A+, A, A-)

Demonstrates a thorough understanding of the concepts in Functional Analysis, and can always apply these to solve a range of mathematical problems

Good (B+, B, B-)

Demonstrates a substantial understanding of the concepts in Functional Analysis, and can usually apply these to solve some mathematical problems

Fair (C+, C, C-)

Demonstrates a general understanding of the concepts in Functional Analysis, and can sometimes apply these to solve some mathematical problems

Marginal (D)

Demonstrates a partial understanding of the concepts in Functional Analysis, and can rarely apply these

Failure (F)

Demonstrates little understanding or misunderstanding of the concepts in Functional Analysis, and can almost never apply these

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Assessment Task

2. Quizzes

Criterion

Ability to understand the basic concepts and applications of Functional Analysis

Excellent (A+, A, A-)

Demonstrates a thorough understanding of the concepts in Functional Analysis, and can always apply these to solve a range of mathematical problems

Good (B+, B, B-)

Demonstrates a substantial understanding of the concepts in Functional Analysis, and can usually apply these to solve some mathematical problems

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Failure (F)

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Assessment Task

3. Formative take-home assignments

Criterion

Ability to understand the basic concepts and applications of Functional Analysis

Excellent (A+, A, A-)

Demonstrates a thorough understanding of the concepts in Functional Analysis, and can always apply these to solve a range of mathematical problems

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Assessment Task

4. Examination

Criterion

Ability to understand the basic concepts and applications of Functional Analysis

Excellent (A+, A, A-)

Demonstrates a thorough understanding of the concepts in Functional Analysis, and can always apply these to solve a range of mathematical problems

Good (B+, B, B-)

Demonstrates a substantial understanding of the concepts in Functional Analysis, and can usually apply these to solve some mathematical problems

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Part III Other Information

Keyword Syllabus

Hilbert and Banach spaces; Linear operators; L2-integrals; Fourier transformation; Spectral theorem; Applications to Quantum Mechanics and partial differential equations (Fredholm Alternative).

Reading List

Compulsory Readings

	Title
1	Lecture Notes

Additional Readings

	Title
1	Brian Hall, Quantum Theory for Mathematicians, Graduate Texts in Mathematics, Springer Verlag
2	Walter Rudin, Real and Complex Analysis, McGraw-Hill Book Company